JOURNAL OF MATERIALS SCIENCE 27 (1992) 3527–3532

Deterministic method for predicting the strength distribution of a fibre bundle

J. J. MASSON
Institute for Materials Research, DLR, D-5000 Köln 90, Germany
E. BOURGAIN
IRSID, Departement Propríetés Démploi, 185 rue du Président Roosevelt, F-78105, St. Germain-en-Laye, France

Owing to the non-strain hardening plastic behaviour of the aluminium matrix and the weak fibre/matrix interface, it has been shown that the strength of a carbon fibre-reinforced aluminium matrix composite made by diffusion bonding of prepreg layers can be derived from the corresponding fibre bundle strength. Application of Coleman's model to predict bundle strength leads to the conclusion that the composite must break when 15% of the fibres are broken. This greatly overestimates the experimental composite strength. Overestimations made by using the Coleman model are due to some implicit assumptions which are not valid in the case under consideration and which may consequently not describe our material. A new approach is proposed for the calculation of the strength distribution of a fibre bundle, based on the same fracture mechanism (fibres fracture progressively until the catastrophic fracture) but without restrictive assumptions. The real interpolated experimental fibre strength distribution (and not the Weibull distribution) is taken into account to predict bundle strength. The proposed method clearly shows the limit of strength prediction, in term of bundle size (number of fibres and gauge length). The risk of making predictions following the Weibull distribution out of the range of the observations (through single-fibre tensile tests) is demonstrated.

1. Introduction

The main objective of this work was to predict the tensile properties of a carbon fibre-reinforced aluminium matrix composite made by hot pressing of prepreg layers from the properties of its components, and to learn how to improve the properties of the components and thus those of the composite. A thorough mechanical characterization of the fibre was carried out to establish how the strength of carbon fibres is affected by reaction with Al during hot pressing. Single fibres were tested in tension after annealing treatments of Al-coated carbon fibres (simulating hot pressing conditions) and dissolution of the Al coating [1]. A new deterministic method of describing rupture probability has been developed and is used to analyse the results of single fibre tensile tests [2].

It is assumed that the tensile properties of the composite mainly depend on the properties of the fibre: i.e. the highest composite strength is obtained with the most resistant fibres. With fibre strength depending on temperature T and time t of the reaction with Al, it is possible to optimize the composite processing parameters T and t, with the additional condition of ensuring sufficient diffusion bonding of the Al-coated carbon fibres during hot pressing.

However, it would also be interesting to be able to predict the exact composite strength from fibres, matrix and – possibly – interface properties. A good understanding of the role and relative importance of the different components on mechanical behaviour is necessary to establish such a prediction model. Thus a thorough characterization of the unidirectional composite was carried out.

The rule of mixtures (ROM) is the most commonly used model to predict composite strength. According to the ROM, the strength of the composite is the mean of matrix and fibre strength weighted by their respective volumic fraction. In this model, fibre and matrix are supposed to undergo the same strain during tensile testing. As an estimation of fibre strength, the middle fibre strength obtained through single-fibre tests is mostly retained, without taking due consideration of the fracture mechanism of the composite. This model considers the variation of strength from fibre to fibre to be negligible, although it can be large and significant (in our case a factor of 4 between the lowest and the greatest failure stress – see Fig. 3a below). Thus, the choice of the fibre strength to be introduced in the ROM equation is not obvious. A first approach consists in calculating the strength of a fibre bundle from fibre-strength distribution and then introducing this strength into the ROM. Coleman [3] applied the theory of bundle strength developed by Daniels [4] to predict the strength of infinite bundles of fibres, which obey the Weibull distribution [5]. The principle of this model is explained more extensively below. The analysis of bundle strength is very useful for determining the characteristic value of fibre strength, but it is not always an accurate representation of failure when the fibres are well bonded with the matrix, and when the matrix is able to redistribute load in the vicinity of a broken fibre through shear stresses at the fibre/matrix interface. In this case, the two models which are mostly used are those of Rosen (cumulative weakening model [6]) and Zweben (fibre break propagation model [7]). A more recent model was developed by Ochiai and co-workers [8, 9] for the case of elastic fibre–plastically deformable metal-matrix composites. In all these models, the fibre strength is assumed to obey a Weibull distribution.

In the case of our C/Al composite material, the mechanical characterization in tensile tests of unidirectional laminate suggests that (i) the matrix behaves plastically without significant strain hardening; and (ii) the fibre/matrix bonding is weak. Therefore the composite strength must be derived from the corresponding bundle strength. Application of Coleman's model leads here to the conclusion that the composite must break when 15% of the fibres are broken. Thus the composite strength must be at least 700 MPa, which is already a large overestimation, although the matrix strength is not taken into account.

It is shown below that very strong assumptions are implicit in Coleman's model, which may consequently not describe a real material. A new approach, which does not need any of these very restrictive assumptions, is proposed to calculate the strength of a bundle, and applied for estimating composite strength.

2. Tensile behaviour of a HM35/Al composite

The composite material is composed of high-modulus carbon fibres (HM35) which reinforce a 99.5 wt % aluminium matrix. Composite unidirectional laminates were manufactured by diffusion bonding of HM35/Al prepreg layers (Toho–Beslon Co., Ltd.). Density measurements and image analysis give an average fibre volume fraction of 35%. Unidirectional laminate test specimens were prepared with a gauge length of 25 mm and a cross-section of 4.13 ± 0.20 mm^2, which correspond to 45 000 ± 2000 fibres in the composite. Tensile tests were performed under a constant displacement rate of 1 mm min^{-1}.

Fig. 1 shows a typical stress–strain curve. Two linear parts and a transition region joining the two parts can be observed. In the first part, the Al matrix

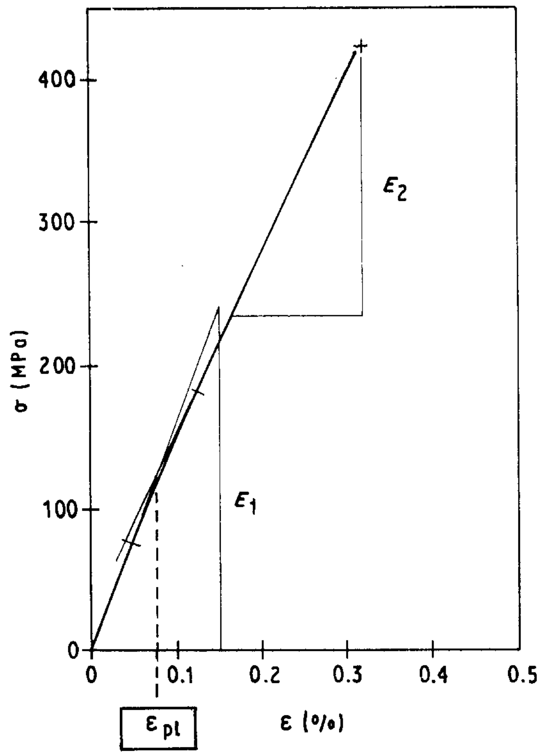

Figure 1 Tensile behaviour of HM35/Al unidirectional composite.

has elastic behaviour; in the second linear part the matrix behaves plastically without strain hardening and therefore makes no more contribution to the E-modulus of the composite. The experimental Young's moduli E_1 (E_2) in the first (second) linear part are in complete agreement with the ROM (see Table I). The matrix yield strain ε_{p1} is 0.06%, whereas the composite failure strain is 0.31 ± 0.07%. The composite strength σ_R is 433 ± 49 MPa.

Thus, according to this behaviour, it can be calculated that only 4% of the load at rupture is carried by the matrix. Moreover, when fibres break the already yielded matrix is not able to carry additional shear stresses. A progressive load transfer from the matrix into the broken fibre by shear stresses at the fibre/matrix interface is therefore not possible. The fibre/matrix bonding is weak, as can be observed on the rupture surface (Fig. 2). Once broken, fibres become ineffective for carrying load and the composite strength can be derived from the corresponding fibre bundle strength.

To calculate bundle strength, the first step is a thorough characterization of single-fibre strength.

3. Mechanical characterization of HM35 single fibres

Since HM35 carbon fibres react with aluminium

TABLE I Prediction of E-moduli through the rule of mixture

Composites	E (GPa)	Volume fraction (%)	$E \times V$ (GPa)	
HM35 fibres	$E_f = 350$*	$V_f = 0.35$		
Al matrix	$E_m = 45$	$V_m = 0.65$	$E_m \times V_m = 29$	$E_f \times V_f = 121$
			$E_f \times V_f + E_m \times V_m$	$E_f \times V_f$
Experimental values:			$E_1 = 150$	$E_2 = 121$

* Akzo Co. data.

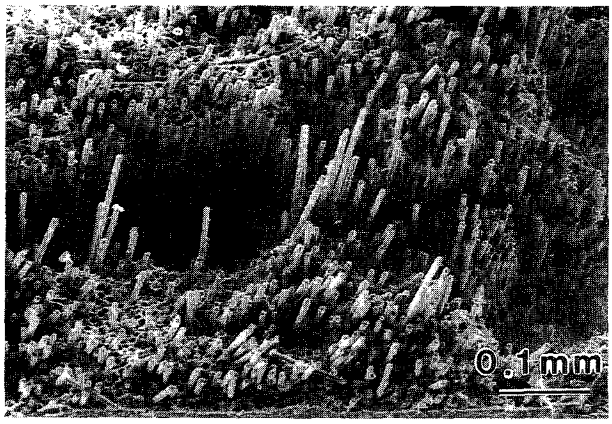

Figure 2 SEM micrograph of rupture surface of HM35/Al composite.

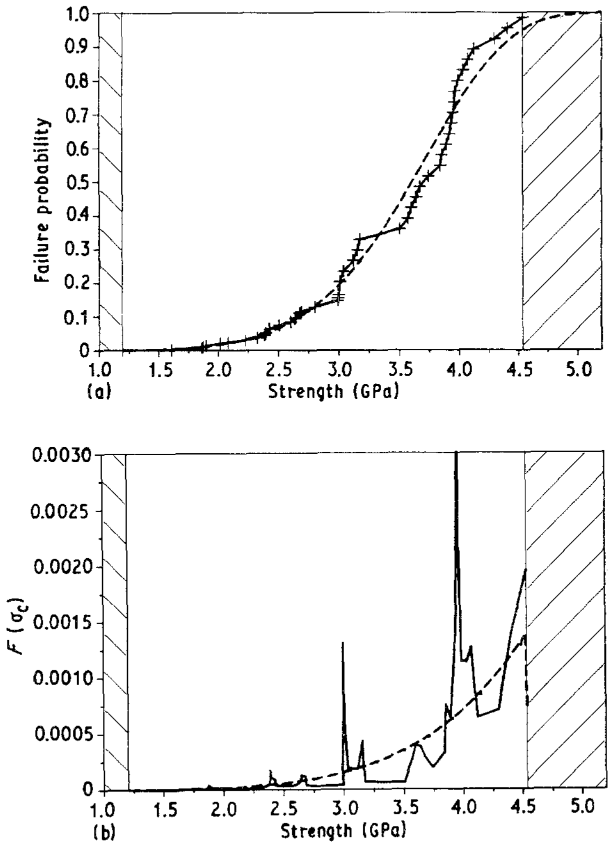

Figure 3 (a) Strength distribution of HM35 fibre after reaction with Al matrix; (b) corresponding defect distribution. (——) Experimental; (- - -) Weibull distribution; $L = 2.5$ mm.

during the consolidation process, single-fibre tests were performed after annealing treatment of Al-coated fibres – which simulates hot pressing conditions – followed by dissolution of the Al matrix [1]. A new method of describing failure probability has been developed [2] and used to analyse the results. It is based on a precise description of the population of critical defects in the fibre, characterized by their critical stress. This model allows extremely precise predictions of experimental strength distributions at gauge lengths other than the experimental lengths, but only in the range of observed strengths. To observe the highest number of defects and to have the broadest prediction range, the most sparing method is to do tensile tests at different complementary gauge lengths. Thus, strength measurements were performed at three different gauge lengths: 2.5, 20 and 40 mm. Putting together the three complementary parts of experimental strength distribution calculated at the smallest of the three gauge lengths L (= 2.5 mm), a "widest" strength distribution could be determined (Fig. 3a). Fig. 3b shows the corresponding defect distribution. This distribution was obtained with three series of 30 tensile tests. About 600 tests at $L = 2.5$ mm would have been necessary to reach the same precision in the description of the material.

4. Classical prediction of the strength of a fibre bundle

Coleman [3] calculated the strength of a fibre bundle tested in tension under strain control (i.e. the strain and thus the stress in the unbroken fibres of the bundle increase at a constant rate). During the tensile test, the load carried by the bundle increases as the load carried by each intact fibre increases in a greater extent more than the number of remaining intact fibres decreases. Just before rupture, this proportion of intact effective fibres decreases catastrophically.

Thus the strength of the bundle is obtained by maximizing the total load carried by the bundle as a function of strength in an unbroken fibre. Assuming that the strength of a fibre obeys a Weibull distribution, the strength of a bundle of fibres σ_b is expressed as follows

$$\sigma_b = \sigma_0 m^{-1/m} \exp(-1/m) \quad (1)$$

where σ_0 is the scale parameter (average strength) and m the shape parameter (characterizing the scatter) determined by fitting experimental strength distributions of fibres with a Weibull function. The fibre cumulative failure probability P_f corresponding to the rupture of the bundle is given by

$$P_f = 1 - \exp(-1/m) \quad (2)$$

Taking $m = 6$, we obtain $P_f = 15\%$, which leads to an overestimation of composite strength.

Three strong assumptions are implicit in this model.

1. The Weibull statistic enables a good description of the material, and the correct Weibull parameters m and σ_0 have been determined.
2. The bundle has an infinite number of fibres, which ensures the continuous stress redistribution in intact fibres after some fibres have broken.
3. The fibres of the bundle break exactly in the order and at the level fixed by the Weibull distribution.

In practice, there is only a finite number of fibres in a bundle. Consequently the strength distribution of these fibres does not exactly follow the Weibull distribution, as already observed in the case of sets of about 30 fibres tested to establish the Weibull distribution itself. Overestimations made by this model are then naturally due to these very restrictive assumptions. Therefore a new approach is proposed based on the previously described fracture mechanisms (progressive fracture until catastrophic fracture) but without any other assumptions.

5. A new approach to predict the strength of a fibre bundle

A bundle of N fibres will be broken at a stress σ if at least one among the N fibres is broken at σ. Considering the stress redistribution, at least one of the $N-1$ remaining intact fibres is broken at $(N/N-1)\,\sigma$ .... and so forth to the last fibre, which must be broken at the stress $N\sigma$. Thus the cumulative rupture probability P_r^b of a bundle of N fibres at stress σ, can be expressed with the rupture probability function of the fibres P_r^f:

$$P_r^b(\sigma) = \prod_{i=0}^{N-1}\left\{1-\left[1-P_r^f\left(\frac{N}{N-i}\,\sigma\right)\right]^{N-i}\right\} \quad (3)$$

For large values of N, this expression can be simplified as

$$P_r^b(\sigma) = \exp\left\{\int_1^N \ln[1-[1-P_r^f(u\,\sigma)]^{N/u}]\,\frac{N}{u^2}\,du\right\} \quad (4)$$

Fig. 4 shows the predicted bundle strength distribution obtained with this model (from the experimental fibre strength distribution represented in Fig. 3a) for different numbers of fibres in the bundle and two gauge lengths (2.5 and 40 mm). The interpolated experimental fibre strength distribution function is taken as P_r^f. No hypothesis is made on the form of the distribution outside the range of observed strength values. As a result, the prediction of the bundle strength can only be made in this range.

The scattering and the average strength of the bundle strength distributions decrease with increasing number of fibres. For $L = 2.5$ mm, a bundle of 20 000 or more is already broken at the smallest strength values observed through single-fibre tensile testing. For $L = 40$ mm, we have little information in the range of small failure probabilities for the single fibre, as it is the largest tested gauge length (at this length it is as if only 34 fibres have been tested, whereas at 2.5 mm it is as if about 600 fibres have been tested). Therefore, for $N > 1000$, a bundle must already be broken at the smallest observed strength and in the range of observed strength, P_r^b is constant (= 1).

It is interesting to know the necessary minimal number n of tested single fibres allowing us to predict the strength distribution of a bundle of N fibres at the same gauge length. Numerical simulations have been made, assuming that the fibres follow a Weibull strength distribution. Results are shown in Fig. 5a for different m-parameters ($m = 1, 3, 6, 9$), i.e. for different scatterings in single fibre strength. The curves give the

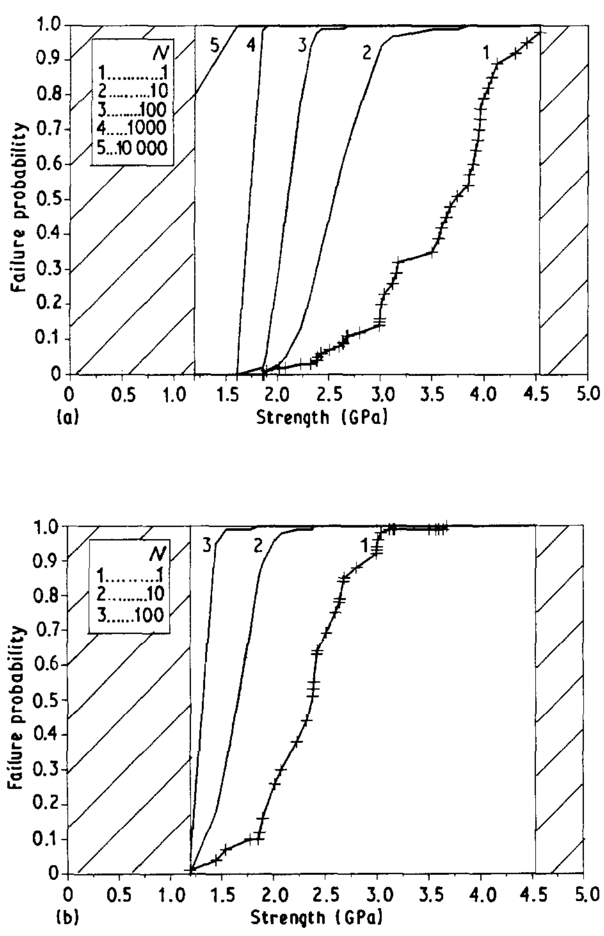

Figure 4 Bundle strength distribution as a function of the number of fibres at a gauge length L of (a) 2.5 and (b) 40 mm. +, Experimental distribution; ——, predictions.

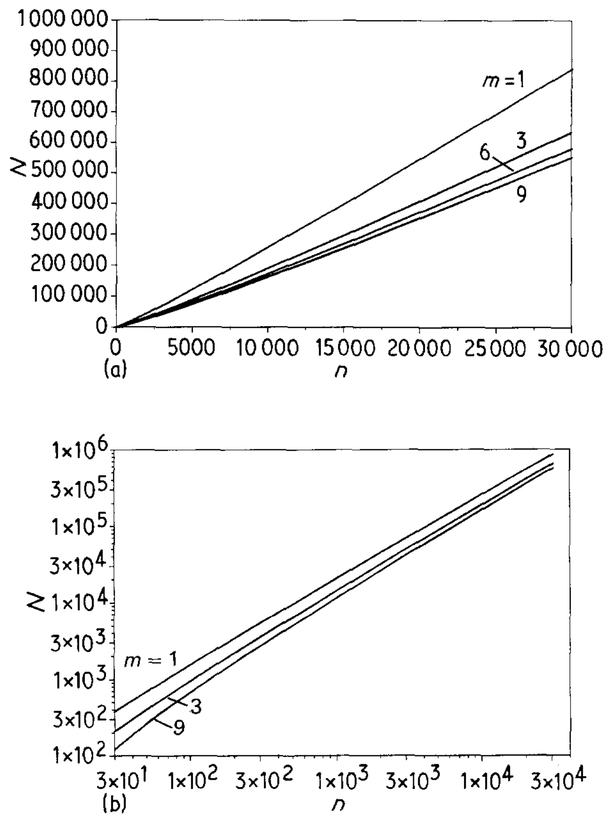

Figure 5 Maximal number N of fibres in a bundle with a failure probability no greater than 50% at the smallest observed strength value, as a function of the number n of tested fibres. (a) Linear; (b) logarithmic scales.

maximal number N of fibres in a bundle as a function of the number n of tested fibres, so that the failure probability of the bundle remains less than 50% for a stress equivalent to the smallest fibre strength observed through n tensile tests.

The more fibres tested, the lower the failure probability at minimal strength which can be observed, and therefore the higher the possible number N of fibres in a bundle without breaking at this smallest strength value. In the case of the C/Al tested fibres, the Weibull shape parameter is about 6. According to the curves of Fig. 5, at the gauge length $L = 40$ mm ($n = 31$) ($L = 2.5$ mm, $n \approx 600$), the largest bundle whose strength distribution can be predicted must contain about 180 (7000) fibres.

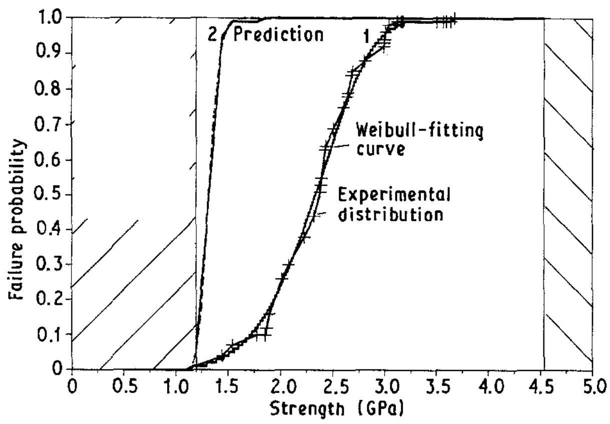

Figure 6 Comparison of bundle prediction in prediction range made from Weibull-fitting curve and from experimental distribution. $L = 40$ mm. Prediction ---, after Weibull-fitting curve; ——, after experimental distribution. N = (1) 1; (2) 100.

6. Prediction of the strength of a C/Al composite

According to the previous study, the failure probability of a bundle of 45 000 fibres is already 1 at the smallest observed fibre strength value. In addition, the scatter of the bundle-strength distribution must be very small and may be neglected. It must be assumed that the strength of the 45 000-fibre bundle is less than or equal to the smallest measured fibre strength, i.e. 1194 MPa.

As the number N of fibres in the bundle is great enough, no information (jumps in the strength distribution, for instance) is lost in predicting the bundle strength distribution from the Weibull fitting instead of the experimental distribution of fibre strength (Fig. 6). It might be expected that the prediction of the 45 000-fibre bundle strength from the Weibull correlation (in this case, extrapolation at strength values lower than the lowest observed fibre strength value) is correct. The results of calculations are summarized in Fig. 7, and leads to a C/Al composite strength between 190 and 260 MPa (depending on the gauge length), which is much lower than the measured value.

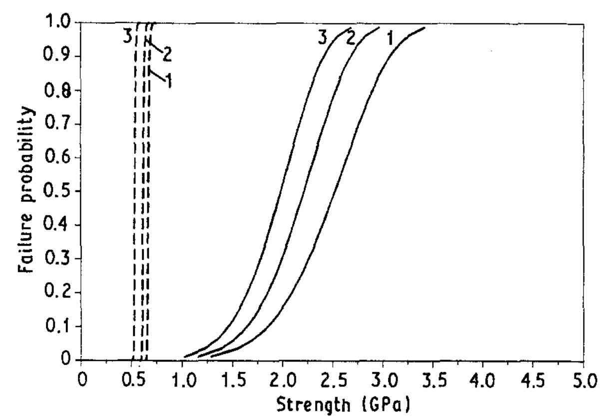

Figure 7 Bundle strength distribution at different gauge lengths calculated from fibre strength distribution following the Weibull statistic (fitted curves). ——, Single fibres ($N = 1$); ---, bundles ($N = 50000$). L = (1) 25; (2) 55; (3) 110 mm.

The risk of making out-of-range predictions following the Weibull statistics has already been mentioned. No critical defect has been observed experimentally under 1194 MPa, and the last failure predictions are implicitly based on the assumption that the fibres may contain critical defects under 600 MPa. Since the predictions are widely underestimated, it may be assumed that there is not so much critical defect in the fibre.

Finally, it has not proved possible to give better than an overestimated value for bundle strength. It corresponds to the most critical observed defect in the fibres. The ROM leads to the C/Al composite strength

$$\sigma_c = 0.35 \times 1194 + 0.65 \times 27 = 435 \text{ MPa}$$

where 27 is the matrix yield strength. 435 MPa is exactly the experimental strength. Thus there is no more critical defect in the fibre than the most critical defect observed through single-fibre tensile tests.

7. Conclusions

The method proposed here for predicting the probability of a bundle is deterministic, as the real composition of the bundle is taken into account to predict its strength. The probability that several fibres in the bundle have the same strength is not zero, and may even be significant.

Considering the real critical defect population of the fibre (Fig. 3b), this probability must be significant, which would cause a more catastrophic rupture of the bundle than is predicted with the classical model.

The proposed method clearly shows the limit of strength predictions which can be made from fibre characterization, in terms of bundle size (length and number of fibres). The problem here is of the same nature as for prediction of the effect of size on fibre strength [2].

Acknowledgements

The authors are grateful to Mr Parenteau (SEP, Bordeaux) for his kind encouragement in the preparation of this work. Thanks are also due to Dr K. Schulte and Professor H. Nowack for helpful discussions. Additionally thanks go to the SEP (Bordeaux) and to the European Community (Contract No. MA1E/0063-C) for funding this work.

References

1. J. J. MASSON, K. SCHULTE, F. GIROT and Y. LEPETIT-CORPS, *Mater. Sci. Engng* **A135** (1991) 59.
2. E. BOURGAIN and J. J. MASSON, *J. Mater. Sci.* (1991) to be published.
3. B. D. COLEMAN, *J. Mech. Phys. Solids* **7** (1958) 60.
4. H. E. DANIELS, *Proc. Roy. Soc. Lond.* **183-A** (1945) 405.
5. W. WEIBULL, *J. Appl. Mech.* **18** (1951) 293.
6. B. W. ROSEN. *AIAA J.* **8** (1964) 1985.
7. C. ZWEBEN, *ibid.* **6** (1968) 2325.
8. S. OCHIAI, K. ABE, and K. OSAMURA, *Z. Metallkde.* **76** (1985) 299.
9. S. OCHIAI, and K. OSAMURA, *ibid.* **76** (1985) 485.

Received 22 February
and accepted 1 July 1991